

WASC 2240

Commentary on
Arithmetic in
Frederick Douglass
Treatise

Appendix 1 UNDERSTANDING FREDERICK DRAYSON'S SUMS

Frederick Drayson produced 18 Tables of detailed costs, so detailed that they became saddled with meaningless precision, but we need to understand what the young man was doing. He costed various items in the production of the three ingredients of gunpowder. Generally he gave a total estimated cost of an item, he gave the tonnage which a particular facility could produce annually, so he arrived at the cost per ton, occasionally he said how many barrels of gunpowder were involved, then he calculated the cost per barrel (his bbl) of gunpowder.

In Table 1, he considered the cost of producing 320 tons of saltpetre. Considering the first item of £164, we can divide that by 320 to get the cost per ton of 10/3d. We cannot proceed further until we understand what the seven figures, given as '4732142+', mean; they must be some sort of 'residual' after a long division has been carried out to an unwarranted precision. When we are able to understand the figures we shall be able to work backwards, to establish the number of barrels involved, where Frederick did not state this.

For Table 2 more information was given. In this case of charcoal, 67 tons was sufficient to produce 10,000 barrels of gunpowder. Here, although there are less figures quoted in the residuals, we can begin to understand what they mean. For the first item of £120, or 120 x 240, or 28,800 pence, we can proceed with the long division of 28,800 by 10,000, which simplifies to 288 by 100. We can see immediately that this reduces to 2.88 pence, which itself is 2¾, or 2.75, plus a 'residual' of 0.13; one might recognise at this stage that 0.13 of a pence, is 0.52, or 4 times 0.13, of a farthing, and this figure is the one quoted as '52 by Frederick; it is the decimal part, or 0.52, of a farthing. It is instructive to see how Frederick would have achieved this by long division (this was first recognised by Chris Sumner):

$$\begin{array}{r}
 2 \frac{3}{4} \\
 100 \overline{) 288} \\
 \underline{200} \\
 88 \times 4 \\
 = 352 \\
 \underline{300} \\
 52
 \end{array}$$

this is the result which emerges
288 pence is to be divided by 100
100 x 2, to give 2 in the result
a remainder of 88 pence x 4
to convert to 352 farthings
100 x 3, to give ¾ in the result
the residual of 52/100, or 0.52 of a farthing

So the cost of the £164 item per barrel of gunpowder was 2¾ pence + 0.52 of a farthing. In this case of the charcoal, where 67 tons was sufficient for 10,000 barrels, we can easily work out the amount of charcoal per barrel, as 67 x 2,240 / 10,000, which equals 15 lb. Since 15 is the percentage of charcoal in the gunpowder mix, it looks possible, even likely, that one of Frederick's barrels will be found to contain 100 lb of gunpowder, but he does not seem to say so directly; rather he states it to be 90lb, apparently in error.

In the case of sulphur there are two versions of the costing: firstly, are the costs, in Table 3, for producing 110 tons of 'refined sulphur' by the process of fusion; secondly, are the costs of refining by the process of sublimation, where the result is either flowers of sulphur (Table 4), or rock sulphur (Table 5); the costs for producing 'about 30 tons' of flowers of sulphur and of 50 tons of rock sulphur are given.

Thus there are five Tables of costs for preparing the ingredients of the gunpowder. Then there are Tables 6 to 17, which concern the subsequent processing. In Table 6, the costs are given for separately grinding the ingredients for 8,000 barrels of gunpowder; this was done in a composition mill. In Table 7, the costs of erecting a mixing house to mix 30,000 barrels of composition are given. In Tables 8 and 9, the costs of amalgamating or incorporating the ingredients of the gunpowder are given; this was done in an incorporating mill, or just called a gunpowder mill; Table 8 is for the mill working day and night, when it was capable of incorporating 1,512 barrels, Table 9 is for the mill working in

the day only, when it could process 648 barrels. It is the extra information about the number of barrels, which we will come back to, in order to demonstrate Frederick's liking for manic long division.

The next operation of converting the mill cake, from the incorporating mill, into grain, the process of corning, was carried out in two different ways. In Table 10, are given the costs using shaking frames, and in Table 11, are the costs using William Congreve's new machine. In Table 12, are costs of dusting the fine grain are given, and in Table 13, of glazing the fine grain. Next the gunpowder has to be dried, this is accomplished in either of two ways: in Table 14, are the costs of drying in a steam stove, and in Table 15, of drying by the use of cast iron pots or in gloom stoves. In Table 16, Frederick summarises the total expense of manufacturing one barrel of gunpowder, the contents of which he states weighed 90lb, but the expectation is that this is a mistake for 100lb. Finally, Table 17 concerns the costs of re-working or regenerating damaged powder.

In the case of most of the items in the above 17 Tables, we meet the residuals given to seven significant figures such as '3968253+; we have learnt that this means Frederick was working to the nearest ten millioneth part of a farthing. Such apparant precision, when working from estimated data in the first place, shows something lacking in the mind of the young Frederick. This particular residual is seen in Table 8, where the repairs to the incorporating mill, producing 1,512 barrels of gunpowder annually, were estimated to cost £40. We will follow the long division which Frederick must have undertaken to produce this result; £40, or 40 x 240, or 9,600 pence, is to be divided by 1,512 to produce the cost per barrel of gunpowder:

OR

$$6 + \frac{1}{4} (1.3968253)$$

$$1512 \overline{) 9600}$$

$$\underline{9072}$$

$$528 \times 4$$

$$= 2112$$

$$\underline{1512}$$

$$6000$$

$$\underline{4536}$$

$$14600$$

$$\underline{13608}$$

$$10320$$

$$\underline{9072}$$

$$12480$$

$$\underline{12096}$$

$$3840$$

$$\underline{3024}$$

$$8160$$

$$\underline{7560}$$

$$6000$$

$$\underline{4536}$$

9,600 pence is to be divided by 1,512
 1,512 times 6, gives the 6 in the result
 this is the remainder x 4
 to convert to farthings
 1,512 x 1, gives the ¼
 remainder 600, carry down 0
 1,512 times 3, gives the 0-3
 remainder 1460, carry 0
 1,512 times 9, gives the 0-09
 remainder 1032, carry 0
 1,512 times 6, gives the 0-006
 remainder 1248, carry 0
 1,512 times 8, gives the 0-0008
 remainder 384, carry 0
 1,512 times 2, gives the 0-00002
 remainder 816, carry 0
 1,512 times 5, gives the 0-000005
 remainder 600, carry 0
 1,512 times 3, gives 0-0000003

So it could go on. The resulting cost of this item per barrel of gunpowder is $6\frac{1}{4} + \frac{1}{4} \times 0.3968253$, or $6 + \frac{1}{4} \times (1 + 0.3968253)$, or 6.3492063, pence; but how ludicrous to take the calculations to such unwarranted precision.

With the reader now fully aware of what young Frederick was up to, we can go back and try to work out how much saltpetre and sulphur were contained in a barrel of gunpowder. Using Frederick's data for charcoal, it was clear that there was 15lb per barrel. From Table 1, for saltpetre, the cost of the first item is £164 for 320 tons, or 10/3d, or 123 pence, per ton. This gives $123 / 2,240$, or 0.549107, pence per lb. The cost per barrel of gunpowder, we can now interpret as $4 + \frac{1}{4} \times 0.4732142$, or 4.1180355 pence. Dividing the cost per barrel by the cost per lb, or 4.1180355 by 0.0549107, as

expected, yields a weight of 75lb of saltpetre per barrel. For sulphur, from Table 3, the first item of £40 for 110 tons, yields a cost of 0.038961 pence per lb. We can interpret the cost of a barrel of sulphur as $\frac{1}{4} + \frac{1}{4} \times 0.5584415$, or $\frac{1}{4} \times 1.5584415$, or 0.3896103, pence. Dividing these two numbers, 0.3896103 by 0.038961, we can see that one barrel of gunpowder contained 10lb of sulphur. These results confirm that Frederick was considering a barrel to contain 100lb, not 90lb as stated, and that the proportions were:

saltpetre	75lb
charcoal	15lb
sulphur	10lb

These few calculations should be sufficient to give the reader confidence to get into the mind of the young apprentice. One may be impressed by the ability of the man, but appalled at his predilection for long division and his obsession with unwarranted precision. Perhaps this confirms that the work is that of an apprentice and not that of a mature engineer.

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